Proportional areas

by Kerstin Fröberg

As I was putting in a patterned wood floor this summer, I was struck anew with how big an area a seemingly narrow border can come to. This has been turning over in my mind, and I decided to explore its patterning possibilities:
what if I made a pattern of rectangles within each other, the areas, heigths and widths of the same proportion to each other?

Let the proportion between heigth and width be K: $K = \frac{h}{w}$

This makes the area $A_0 = h_0 * w_0 = w_0{}^2 * K$
As I wanted the proportions between length and width remain the same, *K* will remain the same.
Therefore, the second area A_1 can be expressed as $w_1{}^2 * K$

If the proportion between A_0 and A_1 is *P*, then $P = \frac{w_0^2 * K}{w_1^2 * K}$

which gives $w_1 = \sqrt{\frac{1}{P}} * w_0$ and $h_1 = w_1 * K$

To begin the play, I started with some easy figures: say that the smallest area is 1 unit wide and 2 units tall. To make things more thread-friendly I multiplied by 10, and rounded the result to nearest even figure.

proportion between areas	width	scaled width	height	scaled height
$A_0 / A_1 = 1 / 1{,}5$	$w_0 = 1$	10	$h_0 = 2$	20
	$w_1 = 1{,}22$	12	$h_1 = 2{,}45$	24
	$w_2 = 1{,}5$	16	$h_2 = 3$	30
	$w_3 = 1{,}84$	18	$h_3 = 3{,}67$	36
	$w_4 = 2{,}24$	22	$h_4 = 4{,}48$	44
$A_0 / A_1 = 1 / 2$	$w_0 = 1$	10	$h_0 = 2$	20
	$w_1 = 1{,}41$	14	$h_1 = 2{,}82$	28
	$w_2 = 2$	20	$h_2 = 4$	40
	$w_3 = 2{,}82$	28	$h_3 = 5{,}65$	56
	$w_4 = 3{,}98$	40	$h_4 = 7{,}97$	80
$A_0 / A_1 = 1 / 3$	$w_0 = 1$	10	$h_0 = 2$	20
	$w_1 = 1{,}73$	18	$h_1 = 3{,}46$	34
	$w_2 = 3$	30	$h_2 = 6$	60
	$w_3 = 5{,}19$	52	$h_3 = 10{,}38$	104
	$w_4 = 8{,}98$	90	$h_4 = 17{,}97$	180

Some figures with a nearly golden rectangle as the basis - $K = \frac{5}{3} = 1{,}67$, scaling factor 3 :

proportion between areas	width	scaled width	height	scaled height
$A_0 / A_1 = 1/2$	$w_0 = 3$	9	$h_0 = 5$	15
	$w_1 = 4{,}24$	13	$h_1 = 7{,}07$	21
	$w_2 = 6$	18	$h_2 = 10{,}02$	30
	$w_3 = 8{,}49$	25	$h_3 = 14{,}18$	43
	$w_4 = 12$	36	$h_4 = 20{,}04$	60
$A_0 / A_1 = 1/3$	$w_0 = 3$	9	$h_0 = 5$	15
	$w_1 = 5{,}20$	16	$h_1 = 8{,}68$	26
	$w_2 = 8{,}99$	27	$h_2 = 15{,}01$	45
	$w_3 = 15{,}57$	47	$h_3 = 26{,}00$	78
	$w_4 = 26{,}97$	81	$h_4 = 45{,}04$	135

I made a number of block drafts based on these figures. I soon found out that although concentric rectangles are all right for a floor, it made boring surface patterns.
I started to offset the inner rectangles both randomly and orderly, let them share one corner…

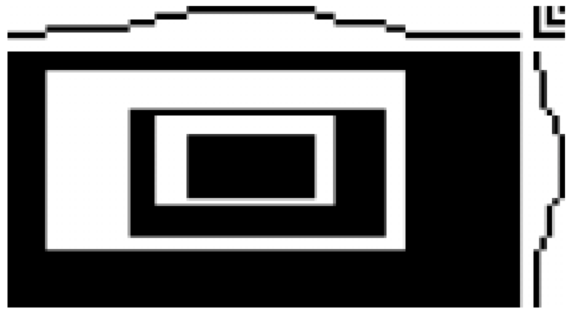
Here are some of my results:

$K = 1/2$, $P = 1/1{,}5$

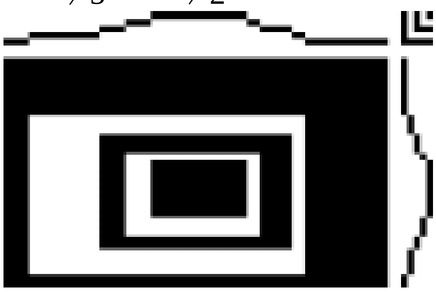
This was not very interesting - it may be better with a bigger scaling factor. This small factor combined with my desire to round to an even figure made the steps too similar to my taste.

$K = 1/2, \; P = 1/2$

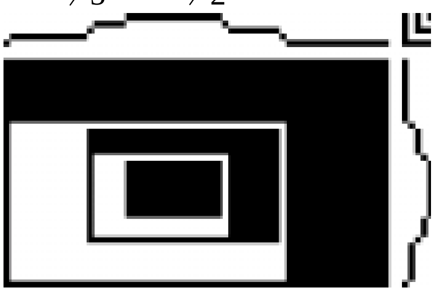
$K = 1/2, \; P = 1/2$, offset randomly



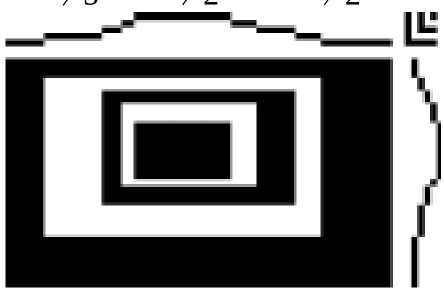
$K = 3/5, \; P = 1/2$, offset randomly



$K = 3/5, \; P = 1/2$, offset 1



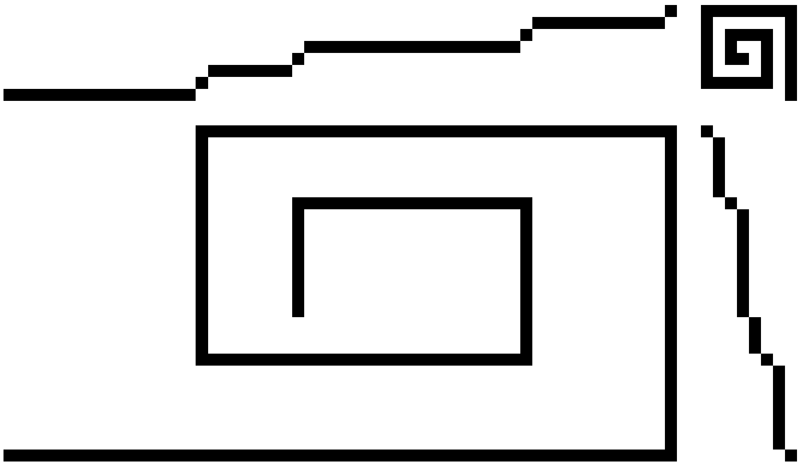
$K = 3/5, \; P = 1/2$, offset $1/2$



$K = 3/5, \; P = 1/3$

Then I went totally overboard and started to do spirals...

Based on $K = \frac{1}{2}$, $P = \frac{1}{2}$:



I could go on...

All in all, this was an interesting excercise, one that may well find its way into some interior fabrics some day. I may use the actual proportions of a door to make draperies, for instance. Or it might be used for placemats, indicating the "correct" positions of the plates and the silverware. Maybe the spiral could be elaborated into a labyrinth for sofa cushions...